

Back reaction, covariant anomaly and effective action

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ABSTRACT: In the presence of back reaction, we first produce the one-loop corrections for the event horizon and Hawking temperature of the Reissner-Nordström black hole. Then, based on the covariant anomaly cancelation method and the effective action technique, the modified expressions for the fluxes of gauge current and energy momentum tensor, due to the effect of back reaction, are obtained. The results are consistent with the Hawking fluxes of a (1+1)-dimensional blackbody at the temperature with quantum corrections, thus confirming the robustness of the covariant anomaly cancelation method and the effective action technique for black holes with back reaction.

KEYWORDS: Black holes, Modes of Quantum Gravity.

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1. Introduction

Hawking proved that black holes are not perfectly black, rather they can radiate particles characterized by the thermal spectrum with the temperature $T = (1/2\pi)\kappa$, where κ is the surface gravity of the black hole[1]. Since then, the study of Hawking radiation has long been attracted a lot of attentions of theoretical physicists. Recently, Robinson and Wilczek have proposed a particularly illuminating way to understand Hawking effect via gravitational anomaly[2]. In this paper, the effective two dimensional theory is formulated outside the horizon to exclude the classically irrelevant ingoing modes, as a result, gravitational anomaly with respect to diffeomorphism symmetry appears. Unitarity is, of course, preserved, so gravitational anomaly should be cancelled by quantum effects of the modes that were irrelevant classically. The result shows that the compensating flux to cancel gravitational anomaly at the horizon is equal to that of (1+1)-dimensional blackbody at the Hawking temperature. Later on, the method is widely used to include gauge anomaly for the charged or rotating black holes[3, 4, 5, 6, 7, 8, 9]. In these observations[2, 3, 4, 5, 6, 7, 8, 9], the Hawking fluxes are derived by requiring both the *consistent* gauge and gravitational anomalies and the vanishing conditions of *covariant* current and energy momentum tensor. To simplify this consistent anomaly model, Banerjee and Kulkarni proposed a more economical and conceptually cleaner model, called as covariant anomaly, to derive Hawking radiation of black holes[10]. Here, the Hawking fluxes can equally well be obtainable from the *covariant* expression only. Further applications of this approach can be found in Refs.[11, 12].

Just at the same time, another effective method to correctly reproduce Hawking fluxes is also proposed to base on the effective action by using the *covariant* boundary condition and the *covariant* energy-momentum tensor[12, 13]. This approach is particularly direct

and useful since only exploitation of known structure of effective action at the horizon is sufficient to determine the Hawking fluxes of energy-momentum tensor.

Based on these developments for the covariant anomaly cancelation method and the effective action technique, some elaborate applications soon appeared in Refs.[10, 11, 12, 13]. None of these computations, however, consider the effect of back reaction. Naturally, it becomes interesting to incorporate the effect of back reaction in these analysis of [10, 11, 12, 13]. Now a question arises whether the covariant anomaly cancelation technique and the effective action method, having been largely proved their robustness, are still applicable to include quantum corrections.

In this paper, starting from the quantum-corrected event horizon and Hawking temperature, we first adopt the covariant anomaly cancelation method to produce the modified Hawking fluxes including the effect of back reaction. The results are consistent with the Hawking fluxes of a (1+1)-dimensional blackbody at the Hawking temperature with quantum corrections, thus confirming the robustness of the covariant anomaly cancelation method for black hole with back reaction. Then, we apply the effective action technique to reproduce the same result, also confirming its robustness for black holes including the effect of back reaction.

The remainders of this paper is organized as follows. In Sec.2, we first reproduce the Reissner-Nordström black hole including the effect of back reaction, and provide the one-loop corrections for its event horizon and Hawking temperature. In the presence of back reaction, Sec.3 and 4 are, respectively, devoted to check the robustness of the covariant anomaly cancelation method and the effective action technique. The modified expressions for the Hawking fluxes, due to the effect of back reaction, are obtained. Sec.5 contains some conclusions and discussions.

2. Back reaction on the Reissner-Nordström black hole

In this section, we aim to study the effects of back reaction on the Reissner-Nordström black hole and from there, the one-loop corrections for its event horizon and Hawking temperature are obtained. For the Reissner-Nordström black hole, the vacuum solution can be written as

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2, \quad (2.1)$$

where $f(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2}$, and the gauge potential $\mathcal{A}_t(r) = -\frac{Q}{r}$. The usual event horizon is located at $r = r_h = m + \sqrt{m^2 - Q^2}$. The usual Hawking temperature is given by $T_h \equiv \frac{\kappa_h}{2\pi} \equiv \frac{1}{4\pi}\partial_r f(r)|_{r_h}$. In the semiclassical approximation, the same results for the event horizon and Hawking temperature can also be reproduced by the tunneling formalism. Beyond the semiclassical approximation, the results would experience higher-order quantum corrections(see in Refs.[14, 15, 16, 17]).

The effect of back reaction, acting as a source of curvature[18], stems from the non-zero vacuum expectation value of the energy momentum tensor. When the matter fields (ϕ) (including the graviton) are quantized at the one-loop level and then coupled to gravity,

the Einstein equation can be written as

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} = \langle T_{\mu\nu}(\phi, g_{\mu\nu}) \rangle, \quad (2.2)$$

By solving Eq.(2.2), and assuming that far away from the black hole we have essentially vacuum, the Reissner-Nordström black hole including the effect of back reaction takes the form as[18]

$$ds_{\text{corr}}^2 = -\mathcal{F}_{\text{corr}}(r)dt^2 + \frac{dr^2}{\mathcal{H}_{\text{corr}}(r)} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2, \quad (2.3)$$

where

$$\begin{aligned} \mathcal{H}_{\text{corr}}(r) &= 1 - \frac{2m}{r} + \frac{Q^2}{r^2} + \frac{1}{r} \int_{r_\infty}^r r^2 T_t^t dr, \\ \mathcal{F}_{\text{corr}}(r) &= \mathcal{H}_{\text{corr}} e^{\int_{r_\infty}^r (T_r^r - T_t^t) r \mathcal{H}_{\text{corr}}^{-1} dr}. \end{aligned} \quad (2.4)$$

Obviously, the corrected terms in (2.4) are related to the non-zero vacuum expectation value of the energy momentum tensor. Here, we use the thermal stress-energy tensor given by Huang [19] for the case of the Reissner-Nordström black hole. As a result, the corrected metric due to back reaction can be fully determined.

Next, we focus on providing the one-loop corrections for the event horizon and Hawking temperature. After carefully analyzing the dimensions and the integrability condition for the first law of thermodynamics of the black hole, the most general term which has the dimension of \hbar can be reformulated as

$$\mathcal{H}_{RN}(m, Q) = a_2 \left(mr_h - \frac{1}{2}Q^2 \right), \quad (2.5)$$

whose concrete derivation appears in Appendix A. Here, a_2 is a dimensionless constant, and $r_h = m + \sqrt{m^2 - Q^2}$ is the usual horizon of the Reissner-Nordström black hole. For the static black hole, the event horizon is determined by solving the equation $g_{tt}(r_+) = 0 = g^{rr}(r_+)$. Under this condition, the modified horizon radius at the one-loop level can then be written as

$$r_+ = r_h \left(1 + \beta \frac{\hbar}{\mathcal{H}_{RN}} \right), \quad (2.6)$$

where

$$\beta = -\frac{\mathcal{H}_{RN}}{\hbar} \frac{1}{2(m - Q^2/r_h)} \int_{r_\infty}^{r_h} r^2 T_t^t dr. \quad (2.7)$$

is a dimensionless constant parameter. Here, the dimensionless constant β can be fully determined since the integral $\int_{r_\infty}^r r^2 T_t^t dr$ is known to us in view of the renormalised energy momentum tensor[19]. Similarly, the one-loop corrections for the surface gravity of the black hole can be read off

$$\kappa_+ = \kappa_h \left(1 + \alpha \frac{\hbar}{\mathcal{H}_{RN}} \right), \quad (2.8)$$

where α is a dimensionless constant and $\kappa_h = \frac{m}{r_h^2} - \frac{Q^2}{r_h^3}$ is the surface gravity of the black hole without the presence of back reaction, and $\kappa_+ = \frac{1}{2} \partial_r \sqrt{\mathcal{F}_{\text{corr}} \mathcal{H}_{\text{corr}}} |_{r_+}$ is the modified surface gravity due to back reaction, and

$$\alpha = \frac{\mathcal{H}_{RN}}{\hbar} \left[\frac{1}{2} \int_{r_\infty}^{r_+} (T_t^t - T_r^r) r \mathcal{H}_{\text{corr}}^{-1} dr + \frac{1}{\kappa_h} \left(r_+ T_t^t(r_+) - \frac{1}{r_+^2} \int_{r_\infty}^{r_+} r^2 T_t^t dr \right) \right]. \quad (2.9)$$

Here, since the integrals $\int_{r_\infty}^r r^2 T_t^t dr$ and $\int_{r_\infty}^r (T_r^r - T_t^t) r \mathcal{H}_{\text{corr}}^{-1} dr$ can be completely determined by the renormalised energy momentum tensor[19], the dimensionless parameter α can also be known to us. Thus, the modified Hawking temperature at the one-loop level can be written as

$$T_{\text{corr}} = \frac{\kappa_+}{2\pi} = \frac{\kappa_h}{2\pi} \left(1 + \alpha \frac{\hbar}{\mathcal{H}_{RN}} \right). \quad (2.10)$$

Till now, as a result of back reaction, we have obtained the one-loop level corrections to the horizon radius and Hawking temperature. It should be noted that β is negative and the effect of back reaction will be a decrease in the usual horizon radius. While the Hawking temperature (2.10) with the one-loop corrections is a perturbative solution, and the constant α is related to the matter-graviton balance. That is, if $\alpha > 0$ (i.e when the matter (scalar field) is dominant), the surface gravity gets enhanced by the effect of back reaction, while for $\alpha < 0$ (i.e when the graviton is dominant), it gets reduced by the one-loop back reaction effect[18]. In fact, the dimensionless constant α is a model-dependent parameter. In the case of Loop Quantum Gravity, α is a negative coefficient whose exact value was once an object of debate but has since been rigorously fixed at $\alpha = -\frac{1}{2}$. In the string theory, the sign of α depends on the number of field species appearing in the low energy approximation.

In the next sections, basing on the covariant anomaly cancelation method and the effective action technique, we focus on deriving the modified Hawking fluxes of the Reissner-Nordström black hole with back reaction .

3. Back reaction and covariant anomalies

As a warm-up, we briefly review the basics idea of applying the covariant anomaly cancelation approach to derive the Hawking fluxes. An anomaly in a quantum field theory is a conflict between a symmetry of the classical action and the procedure of quantization. Since the horizon is a null hypersurface, modes interior to the horizon cannot, classically, affect physics outside the horizon. If we formally remove the classically irrelevant ingoing modes to obtain the effective action in the exterior region, it becomes anomalous with respect to gauge and diffeomorphism symmetries. The underlying theory is, of course, invariant. To restore gauge invariance and general coordinate covariance at the quantum level, one must introduce the fluxes of gauge charge current and energy momentum tensor to cancel gauge and gravitational anomalies at the horizon, which is compatible with the Hawking fluxes of the charge and energy momentum tensor.

In this section, we adopt covariant anomaly cancelation method to derive Hawking radiation of the Reissner-Nordström black hole with back reaction, and expect that the compensating fluxes required to cancel the covariant gauge and gravitational anomalies at the horizon have an equivalent form to those of Hawking radiation with a chemical potential. To complete that, near the black hole horizon, we first introduce a dimensional reduction technique to reduce the higher-dimensional theory to the effective two-dimensional one. For simplicity, we consider the action of a massless scalar field in the black hole background with the minimal electro-magnetic coupling term. Upon introducing the tor-

toise coordinate transformation $\frac{\partial r_*}{\partial r} = \frac{1}{\sqrt{\mathcal{F}_{\text{corr}}(r)\mathcal{H}_{\text{corr}}(r)}}$ and a partial wave decomposition as $\Psi(t, r, \theta, \phi) = Y_{lm}(\theta, \phi)\Psi(t, r)$, the physics near the horizon can be described by an infinite set of (1+1)-dimensional fields in the spacetime with the metric

$$ds^2 = -\mathcal{F}_{\text{corr}}(r)dt^2 + \frac{dr^2}{\mathcal{H}_{\text{corr}}(r)}, \quad (3.1)$$

and the gauge potential $\mathcal{A}_t(r) = -\frac{Q}{r}$. In the two-dimensional reduction, when omitting the classically irrelevant ingoing modes at the horizon, the effective theory contains gauge and gravitational anomalies. To restore gauge invariance and diffeomorphism covariance at the quantum level, we should introduce the fluxes of gauge current and energy momentum tensor to cancel gauge and gravitational anomalies. Let's, first, study gauge anomaly and gauge current flux.

As is well-known that the covariant form for the anomalous Ward identity can be written as $\nabla_\mu J^\mu = -\frac{e^2}{4\pi\sqrt{-g}}\epsilon^{\mu\nu}F_{\mu\nu} = \frac{e^2}{2\pi\sqrt{-g}}\partial_r\mathcal{A}_t(r)$. Near the horizon, the classically irrelevant ingoing modes is excluded to formulate the effective field theory, so the gauge current here exhibits an anomaly with respect to gauge invariance. For the left-handed field, the covariant gauge current satisfies $\partial_r[\sqrt{-g}J_{(H)}^r] = \frac{e^2}{2\pi}\partial_r\mathcal{A}_t(r)$. Away from the horizon, since there is no anomaly, the gauge current is conserved here, and satisfies $\nabla_\mu J_{(o)}^\mu = 0$. Integrating the gauge currents in the two regions yields

$$\begin{aligned} \sqrt{-g}J_{(o)}^r &= c_o, \\ \sqrt{-g}J_{(H)}^r &= c_H + \frac{e^2}{2\pi} \int_{r_+}^r dr \partial_r \mathcal{A}_t(r), \end{aligned} \quad (3.2)$$

where c_o and c_H are integration constants, respectively denoting the covariant gauge currents at $r = \infty$ and $r = r_+$. If introducing two step functions $\Theta_+(r) = \Theta(r - r_+ - \varepsilon)$ and $H(r) = 1 - \Theta_+(r)$, the total current can be written as $J^\mu = J_{(o)}^\mu \Theta_+(r) + J_{(H)}^\mu H(r)$. Then the Ward identity can be rewritten as

$$\partial_r[\sqrt{-g}J^r] = \partial_r\left(\frac{e^2}{2\pi}\mathcal{A}_t(r)H(r)\right) + \left[\sqrt{-g}(J_{(o)}^r - J_{(H)}^r) + \frac{e^2}{2\pi}\mathcal{A}_t(r)\right]\delta(r - r_+ - \varepsilon). \quad (3.3)$$

To make the gauge anomaly free under gauge transformation, the first term should be canceled by the quantum effect of the classically irrelevant ingoing modes, and the second term should vanish at the horizon, which yields

$$c_o = c_H - \frac{e^2}{2\pi}\mathcal{A}_t(r_+). \quad (3.4)$$

Considering the regularity requirement of the physical quantities at the further horizon, we impose the boundary condition that the covariant current vanishes at the horizon, which means $c_H = 0$. This determines the gauge current flux to be

$$c_o = -\frac{e^2}{2\pi}\mathcal{A}_t(r_+) = \frac{e^2 Q}{2\pi r_h} \left(1 + \beta \frac{\hbar}{\mathcal{H}_{RN}}\right)^{-1}. \quad (3.5)$$

This is the modified gauge current flux in the presence of back reaction, whose value is expected to equal that of a (1+1)-dimensional blackbody at the temperature with quantum corrections, as will appear presently. When $\hbar \rightarrow 0$, we can reproduce the usual current flux as Ref.[3].

Apart from gauge anomaly, when excluding the classically irrelevant ingoing modes at the horizon, gravitational anomaly also takes place as a consequence as a breakdown of the diffeomorphism symmetry, which is normally expressed as the nonconservation of the energy momentum tensor. Away from the horizon, there is no anomaly, but contains an effective background gauge potential, so the energy momentum tensor satisfies the modified conservation equation as $\nabla_\mu T_{(o)\nu}^\mu = F_{\mu\nu} J_{(o)}^\mu$. While the energy momentum tensor near the horizon obeys the anomalous Ward identity after adding gravitational anomaly as $\nabla_\mu T_{(H)\nu}^\mu = F_{\mu\nu} J_{(H)}^\mu + A_\nu$, where $A_\nu = -\frac{1}{96\pi}\sqrt{-g}\epsilon_{\mu\nu}\partial^\mu R$. For the metric of the form (3.1), the anomaly is purely timelike, which means $A_r = 0$ and $A_t = \frac{1}{\sqrt{-g}}\partial_r N_t^r(r)$ where $N_t^r(r) = \frac{1}{96\pi}(\mathcal{H}_{\text{corr}}\mathcal{F}_{\text{corr}}'' + \frac{1}{2}\mathcal{H}_{\text{corr}}'\mathcal{F}_{\text{corr}}' - \frac{1}{\mathcal{F}_{\text{corr}}}\mathcal{F}_{\text{corr}}'^2\mathcal{H}_{\text{corr}})$. Now, integrating the energy momentum tensor in both regions, we have

$$\begin{aligned}\sqrt{-g}T_{(o)t}^r &= a_o + c_o\mathcal{A}_t(r), \\ \sqrt{-g}T_{(H)t}^r &= a_H + \int_{r_+}^r dr \partial_r (c_o\mathcal{A}_t(r) + \frac{e^2}{4\pi}\mathcal{A}_t^2(r) + N_t^r(r)),\end{aligned}\quad (3.6)$$

where a_o and a_H are integration constants, respectively representing the values of the energy flow at the infinity and the horizon. Writing the total energy momentum tensor as $T_\nu^\mu = T_{(o)\nu}^\mu\Theta_+(r) + T_{(H)\nu}^\mu H(r)$, we then have

$$\begin{aligned}\sqrt{-g}\nabla_\mu T_t^\mu &= c_o\partial_r\mathcal{A}_t(r) + \partial_r\left[\left(\frac{e^2}{4\pi}\mathcal{A}_t^r(r) + N_t^r\right)H(r)\right] \\ &+ [\sqrt{-g}(T_{(o)t}^r - T_{(H)t}^r) + \frac{1}{4\pi}\mathcal{A}_t^2(r) + N_t^r(r)]\delta(r - r_+ - \varepsilon).\end{aligned}\quad (3.7)$$

In Eq.(3.7), the first term is the classical effect of the background electric field for constant current flow. The second term should be canceled by the quantum effect of the classically irrelevant ingoing modes. In order to restore the diffeomorphism covariance at the quantum level, the third term must vanish at the horizon, which yields

$$a_o = a_H + \frac{e^2}{4\pi}\mathcal{A}_t^2(r_+) - N_t^r(r_+).\quad (3.8)$$

To fix a_o completely, it is necessary to impose the boundary condition that the covariant energy momentum tensor vanishes at the horizon, which means $a_H = 0$. So the total flux of the energy momentum tensor is given by

$$\begin{aligned}a_o &= \frac{e^2}{4\pi}\mathcal{A}_t^2(r_+) - N_t^r(r_+) = \frac{e^2}{4\pi}\mathcal{A}_t^2(r_+) + \frac{1}{192\pi}\mathcal{F}_{\text{corr}}'(r_+)\mathcal{H}_{\text{corr}}'(r_+), \\ &= \frac{e^2}{4\pi}\mathcal{A}_t^2(r_+) + \frac{\pi}{12}T_{\text{corr}}^2 \\ &= \frac{e^2 Q^2}{4\pi r_h^2}\left(1 + \beta\frac{\hbar}{\mathcal{H}_{RN}}\right)^{-2} + \frac{\pi}{12}T_h^2\left(1 + \alpha\frac{\hbar}{\mathcal{H}_{RN}}\right)^2.\end{aligned}\quad (3.9)$$

This is the compensating energy flux for canceling the covariant anomaly at the horizon, whose value is expectantly equal to that of a (1+1)-dimensional blackbody with the temperature undergoing quantum corrections. When setting $\hbar \rightarrow 0$, the usual energy flux, as appears in Ref.[3], can also be reproduced.

Next, we will produce the fluxes from blackbody radiation at the temperature T_{corr} with a chemical potential, so as to compare the results with the compensating fluxes for canceling gauge and gravitational anomalies at the horizon. The Planck distribution for the Reissner-Nordström black hole with back reaction can be written as $\mathcal{N}_{\pm e}(\omega) = \frac{1}{e^{(\omega \pm e\mathcal{A}_t(r_+))/T_{\text{corr}} \pm 1}}$ for bosons and fermions, respectively. Here, we only consider the fermion case without the loss of generality. With the Planck distribution, the fluxes of charge and energy momentum tensor are

$$\begin{aligned} (\text{Flux})_{\text{charge}} &= e \int_0^\infty \frac{d\omega}{2\pi} [\mathcal{N}_e(\omega) - \mathcal{N}_{-e}(\omega)] \\ &= -\frac{e^2}{2\pi} \mathcal{A}_t(r_+) = \frac{e^2 Q}{2\pi r_h} \left(1 + \beta \frac{\hbar}{\mathcal{H}_{RN}}\right)^{-1}, \end{aligned} \quad (3.10)$$

$$\begin{aligned} (\text{Flux})_{\text{energy}} &= \int_0^\infty \frac{d\omega}{2\pi} \omega [\mathcal{N}_e(\omega) + \mathcal{N}_{-e}(\omega)] \\ &= \frac{e^2}{4\pi} \mathcal{A}_t^2(r_+) + \frac{\pi}{12} T_{\text{corr}}^2 \\ &= \frac{e^2 Q^2}{4\pi r_h^2} \left(1 + \beta \frac{\hbar}{\mathcal{H}_{RN}}\right)^{-2} + \frac{\pi}{12} T_h^2 \left(1 + \alpha \frac{\hbar}{\mathcal{H}_{RN}}\right)^2. \end{aligned} \quad (3.11)$$

Obviously, the results (3.5) and (3.9) derived from the covariant anomaly cancelation approach are compatible with the results (3.10) and (3.11). So we can easily find that, even for the black hole with back reaction, the thermal fluxes required by black hole thermodynamics are still capable of canceling the covariant gauge and gravitational anomalies at the horizon, and restoring gauge invariance and general coordinate covariance at the quantum level to hold in the effective theory. Thus we have successfully verified the robustness of the covariant anomaly cancelation method in the presence of back reaction. Based on the same idea, in the next section, we aim to check whether the effective action technique is still applicable to include back reaction.

4. Back reaction and effective action

In this section, we will base solely on the structure of the effective action and the covariant boundary conditions at the horizon to cross-check thermodynamic properties of the black hole with back reaction. As mentioned in Sec.3, with the aid of the dimensional reduction technique, the higher theory near the horizon can be effectively described by the two dimensional chiral theory. For the reduced two-dimensional chiral theory, the expression for the anomalous (chiral) effective action has already been derived [13, 24]. Now taking appropriate functional derivatives of the anomalous effective action yields the anomalous gauge current and energy momentum tensor near the horizon. As a result, some parameters appears to be determined. To accomplish it, we then impose the boundary condition that

the covariant gauge current and energy momentum tensor vanish at the horizon. Once these are fixed, the fluxes of the charge and energy momentum tensor can be obtained by taking the asymptotic ($r \rightarrow \infty$) limits of the anomalous gauge current and energy momentum tensor.

Near the horizon, the chiral effective action can be written as [24]

$$\Gamma_{(H)} = -\frac{1}{3}z(\omega) + z(\mathcal{A}), \quad (4.1)$$

with

$$z(v) = \frac{1}{4\pi} \int d^2x d^2y \epsilon^{\mu\nu} \partial_\mu v_\nu(x) \square^{-1}(x, y) \partial_\rho [(\epsilon^{\rho\sigma} + \sqrt{-g}g^{\rho\sigma})v_\sigma(y)], \quad (4.2)$$

where $\square = \nabla^\mu \nabla_\mu$ is the Laplacian operator in the two-dimensional background, and \mathcal{A}_μ and ω_μ are, respectively, the gauge field and the spin connection. Carrying on a variation for the effective action, we can obtain the gauge current and energy momentum tensor. To get their covariant forms in which we are interested, one needs to add appropriate local polynomials [24]. The final expressions for the covariant gauge current J^μ and energy momentum tensor T_ν^μ can be obtained as

$$J^\mu = -\frac{e^2}{2\pi} D^\mu B, \quad (4.3)$$

$$T_\nu^\mu = \frac{e^2}{4\pi} D^\mu B D_\nu B + \frac{1}{96\pi} \left(\frac{1}{2} D^\mu F D_\nu F - D^\mu D_\nu F + \delta_\nu^\mu R \right), \quad (4.4)$$

where $D_\mu = \nabla_\mu - \sqrt{-g}\epsilon_{\mu\nu}\nabla^\nu = -\sqrt{-g}\epsilon_{\mu\nu}D^\nu$ is the chiral covariant derivative. It should be noted that the anomalous Ward identities can be obtained by taking the covariant divergence of (4.3) and (4.4). Here, the definitions of $B(x)$ and $F(x)$ are provided by

$$\begin{aligned} B(x) &= \int d^2y \sqrt{-g} \square^{-1}(x, y) \epsilon^{\mu\nu} \partial_\mu \mathcal{A}_\nu(y), \\ F(x) &= \int d^2y \square^{-1}(x, y) \sqrt{-g} R(y). \end{aligned} \quad (4.5)$$

which satisfy the equations $\square B = -\partial_r \mathcal{A}_t(r)$ and $\square F = R$. Solving the two equations yield $B = B_o(r) - at + b$ and $F = F_o(r) - 4pt + q$, where $B_o(r)$ and $F_o(r)$ satisfy $\partial_r B_o(r) = \frac{1}{\sqrt{\mathcal{F}_{\text{corr}} \mathcal{H}_{\text{corr}}}} (\mathcal{A}_t(r) + c)$ and $\partial_r F_o(r) = \frac{1}{\sqrt{\mathcal{F}_{\text{corr}} \mathcal{H}_{\text{corr}}}} \left(\sqrt{\frac{\mathcal{H}_{\text{corr}}}{\mathcal{F}_{\text{corr}}}} \mathcal{F}'_{\text{corr}} + z \right)$, in which a, b, c, p, q, z are constants of integration.

Next, based on the expressions for the anomalous gauge current (4.3) and energy momentum tensor (4.4), we focus on calculating the fluxes of gauge charge and energy momentum tensor. First we investigate the gauge charge flux. Considering the $\mu = r$ component of the anomalous covariant gauge current J^μ , we have

$$J^r(r) = \frac{e^2}{2\pi} \sqrt{\frac{\mathcal{H}_{\text{corr}}}{\mathcal{F}_{\text{corr}}}} [\mathcal{A}_t(r) + a + c]. \quad (4.6)$$

To fix the parameters in Eq.(4.6), we impose the boundary condition that the covariant gauge current vanishes at the horizon, which means $J^r(r_+) = 0$. As a result, $a + c = -\mathcal{A}_t(r_+)$. As it is well-known, the charge flux is often determined by the asymptotic

$(r \rightarrow \infty)$ limits of the covariant anomalous free current. In fact, in this limit, we observe that the anomaly vanishes. So the charge flux can be directly obtained by taking the asymptotic limit of Eq.(4.6) multiplied by an overall factor of $\sqrt{-g}$, which yields

$$c_o = (\sqrt{-g}J^r)(r \rightarrow \infty) = -\frac{e^2}{2\pi}\mathcal{A}_t(r_+) = \frac{e^2Q}{2\pi r_h}\left(1 + \beta\frac{\hbar}{\mathcal{H}_{RN}}\right)^{-1}. \quad (4.7)$$

This is the modified charge flux based on the effective action technique, whose value exactly agrees with the charge flux (3.10). So, in the presence of back reaction, the effective action technique can correctly reproduce the modified charge flux of Hawking radiation.

Now we focus our attention on the flux of energy momentum tensor. The $r - t$ component of the anomalous covariant energy momentum tensor (4.4) can be derived as

$$\begin{aligned} T_t^r(r) &= \frac{e^2}{4\pi}\sqrt{\frac{\mathcal{H}_{\text{corr}}}{\mathcal{F}_{\text{corr}}}}[\mathcal{A}_t(r) - \mathcal{A}_t(r_+)]^2 \\ &+ \frac{1}{192\pi}\sqrt{\frac{\mathcal{H}_{\text{corr}}}{\mathcal{F}_{\text{corr}}}}[4p + \sqrt{\frac{\mathcal{H}_{\text{corr}}}{\mathcal{F}_{\text{corr}}}}\mathcal{F}'_{\text{corr}} + z]^2 \\ &- \frac{1}{96\pi}\sqrt{\frac{\mathcal{H}_{\text{corr}}}{\mathcal{F}_{\text{corr}}}}\left[\sqrt{\frac{\mathcal{H}_{\text{corr}}}{\mathcal{F}_{\text{corr}}}}\mathcal{F}'_{\text{corr}}(4p + \sqrt{\frac{\mathcal{H}_{\text{corr}}}{\mathcal{F}_{\text{corr}}}}\mathcal{F}'_{\text{corr}} + z)\right. \\ &\left.+ \mathcal{H}_{\text{corr}}\mathcal{F}''_{\text{corr}} - \frac{1}{2}\mathcal{F}'_{\text{corr}}\left(\frac{\mathcal{H}_{\text{corr}}}{\mathcal{F}_{\text{corr}}}\mathcal{F}'_{\text{corr}} - \mathcal{H}'_{\text{corr}}\right)\right]. \end{aligned} \quad (4.8)$$

At the horizon, implementing the boundary condition that the covariant momentum tensor vanishes at the horizon, namely $T_t^r(r_+) = 0$, yields the equation $4p = z \pm \sqrt{\mathcal{F}'_{\text{corr}}(r_+)\mathcal{H}'_{\text{corr}}(r_+)}$. So the total flux of the energy momentum tensor, which is often given by the asymptotic limit for the anomaly free energy momentum tensor, can be given by

$$\begin{aligned} a_o &= (\sqrt{-g}T_t^r)(r \rightarrow \infty) = \frac{e^2}{4\pi}\mathcal{A}_t^2(r_+) + \frac{1}{192\pi}\mathcal{F}'_{\text{corr}}(r_+)\mathcal{H}'_{\text{corr}}(r_+) \\ &= \frac{e^2}{4\pi}\mathcal{A}_t^2(r_+) + \frac{\pi}{12}T_{\text{corr}}^2 \\ &= \frac{e^2Q^2}{4\pi r_h^2}\left(1 + \beta\frac{\hbar}{\mathcal{H}_{RN}}\right)^{-2} + \frac{\pi}{12}T_h^2\left(1 + \alpha\frac{\hbar}{\mathcal{H}_{RN}}\right)^2. \end{aligned} \quad (4.9)$$

This is the modified flux of energy momentum tensor obtained from the effective action technique, whose value has the same form as that in Eqs.(3.11). So, in the presence of back reaction, the effective action technique can also correctly reproduce the modified energy flux of Hawking radiation. From Eqs.(4.7) and (4.9), we can conclude that the effective action technique is still applicable for the black hole with back reaction.

5. Conclusions and discussions

In this paper, our motivation is to check the robustness of the covariant anomaly cancellation method and the effective action technique in the presence of back reaction. To complete that, we first obtain the modified Reissner-Nordström black hole which captures the effect of back reaction. At the one-loop level, we also produce the modified expressions

for its event horizon and Hawking temperature. Then, in the presence of back reaction, we adopt the covariant anomaly cancelation method and the effective action technique to obtain the modified fluxes of charge and energy momentum tensor. The results are completely consistent with the Hawking fluxes of a (1+1)-dimensional blackbody at the temperature with quantum corrections, thus confirming the robustness of the covariant anomaly cancelation method and the effective action technique for black holes with back reaction. When omitting the effect of back reaction (namely, $\hbar \rightarrow 0$), we can also reproduce the same results as appeared in Ref.[3].

With the aid of the corrected Hawking temperature (2.10), we can proceed with the calculation of the modified Bekenstein-Hawking entropy. The modified form of the first law of thermodynamics for the Reissner-Nordström black hole with back reaction can be written as

$$dS_{\text{bh}} = \frac{1}{T_{\text{corr}}} dm - \frac{\mathcal{A}_t(r_h)}{T_{\text{corr}}} dQ. \quad (5.1)$$

On the premise that the entropy is a state function for the spacetime even in the presence of back reaction, then dS_{bh} is an exact differential. This yields the following relation

$$\left. \frac{\partial}{\partial m} \left(-\frac{\mathcal{A}_t(r_h)}{T_{\text{corr}}} \right) \right|_Q = \left. \frac{\partial}{\partial Q} \left(\frac{1}{T_{\text{corr}}} \right) \right|_m. \quad (5.2)$$

If the condition (5.2) holds, the solution of (5.1) can be written as

$$S_{\text{bh}} = \int \frac{dm}{T_{\text{corr}}} - \int \frac{\mathcal{A}_t(r_h)}{T_{\text{corr}}} dQ - \int \frac{\partial}{\partial Q} \left(\int \frac{dm}{T_{\text{corr}}} \right) dQ. \quad (5.3)$$

Let's first solve the integration over m , which yields

$$\begin{aligned} \int \frac{dm}{T_{\text{corr}}} &= \int \frac{1}{T_h} \left(1 + \sum_i (-\alpha)^i \frac{\hbar^i}{\mathcal{H}_{RN}^i} \right) dm \\ &= \frac{\pi}{\hbar} (2mr_h - Q^2) - 2\pi\hat{\alpha} \log(2mr_h - Q^2) \\ &\quad - \frac{4\pi\hat{\alpha}^2 \hbar}{2mr_h - Q^2} + \text{constant} + \text{higher order terms}. \end{aligned} \quad (5.4)$$

where $\hat{\alpha} = \frac{\alpha}{a_2}$. In view of the result (5.4), we can easily check the following relation

$$\frac{\partial}{\partial Q} \int \frac{dm}{T_{\text{corr}}} = -\frac{\mathcal{A}_t(r_h)}{T_{\text{corr}}}. \quad (5.5)$$

Under this condition (5.5), the entropy of the black hole in presence of back reaction is now given by

$$S_{bh} = S_{BH} - 2\pi\hat{\alpha} \log S_{BH} - \frac{4\pi^2\hat{\alpha}^2}{S_{BH}} + \text{constant} + \text{higher order terms}, \quad (5.6)$$

where $S_{BH} = \frac{\pi}{\hbar} (2mr_h - Q^2)$ is the usual Bekenstein-Hawking entropy for the black hole, and the other terms appear as a result of the presence of back reaction. Obviously, the leading correction is logarithmic while the sub-leading terms involve inverse powers of the

entropy. The similar corrections have been previously presented in field theory methods [20], quantum geometry techniques [21], general statistical mechanical arguments [22] and Cardy formula [23] etc. Here, the dimensionless constant $\hat{\alpha}$ is related to the trace anomaly as [16]

$$\hat{\alpha} = -\frac{1}{4\pi} \text{Im} \int d^4x \sqrt{-g} T_\mu^\mu = \frac{1}{180\pi} \left(1 + \frac{3}{10} \frac{2m^2 - r_h^2}{mr_h - Q^2} \right). \quad (5.7)$$

A. Dimensional analysis

In the (3+1) dimensions, the Plank length $l_p = \sqrt{\frac{\hbar G}{c^3}}$, the Plank mass $m_p = \sqrt{\frac{\hbar c}{G}}$, and the Plank charge $Q_p = \sqrt{4\pi c \hbar \epsilon_0}$. So, in the unit of $G = c = \frac{1}{4\pi\epsilon_0}$, the dimensions of the Plank length, mass and charge all take the same as $\sqrt{\hbar}$. So, the general form for the dimension of \hbar should be constructed, in terms of black hole parameters, as

$$\mathcal{H}_{RN}(m, Q) = a_1 r_h^2 + a_2 m r_h + a_3 m^2 + a_4 r_h Q + a_5 m Q + a_6 Q^2, \quad (A.1)$$

where $a_1, a_2, a_3, a_4, a_5, a_6$ are constants, and $r_h = m + \sqrt{m^2 - Q^2}$ is the usual horizon of the Reissner-Nordström black hole. Noted that the entropy is a state function for all stationary spacetimes even in the presence of quantum correction, which means dS_{bh} must be an exact differential. This condition yields Eq.(5.2), and then leads to

$$\left. \frac{\partial \mathcal{H}_{RN}}{\partial m} \right|_Q = - \frac{1}{\mathcal{A}_t(r_h)} \left. \frac{\partial \mathcal{H}_{RN}}{\partial Q} \right|_m. \quad (A.2)$$

Substituting Eq.(A.1) into Eq.(A.2), we have

$$\begin{aligned} & (2a_1 r_h + a_2 m + a_4 Q) \left(1 + \frac{m}{\sqrt{m^2 - Q^2}} \right) + a_4 r_h + 2a_3 m + a_5 Q \\ &= \frac{r_h}{Q} \left[(2a_1 r_h + a_2 m + a_4 Q) \frac{Q}{\sqrt{m^2 - Q^2}} - a_4 r_h - a_5 m - 2a_6 Q \right]. \end{aligned} \quad (A.3)$$

In Eq.(A.1), there are six undetermined constants, so, to fully determine \mathcal{H}_{RN} , we need five separate equations but only one equation (A.2) is applicable. How can finish that. Supposing the thermodynamic entities at each process for the charged black hole evolving from the non-extremal one to the extremal one ($m = Q$) satisfy the modified first law thermodynamics (5.1), only taking different relation between the mass m and the charge Q . Now, we take three cases:

For I case, when $Q = \frac{\sqrt{3}}{2}m$, Eq.(A.3) can be rewritten as

$$\frac{3}{2}a_2 + 2a_3 + \frac{\sqrt{3}}{2}a_5 = -\frac{3\sqrt{3}}{2}a_4 - \sqrt{3}a_5 - 3a_6. \quad (A.4)$$

For II case, when $Q = \frac{\sqrt{15}}{4}m$, Eq.(A.3) can be read off

$$\frac{5}{4}a_2 + 2a_3 + \frac{\sqrt{15}}{4}a_5 = -\frac{5\sqrt{15}}{12}a_4 - \frac{\sqrt{15}}{3}a_5 - \frac{5}{2}a_6. \quad (A.5)$$

For III case, when $Q = \frac{2\sqrt{2}}{3}m$, Eq.(A.3) is changed as

$$\frac{4}{3}a_2 + 2a_3 + \frac{2\sqrt{2}}{3}a_5 = -\frac{4\sqrt{2}}{3}a_4 - \sqrt{2}a_5 - \frac{8}{3}a_6. \quad (\text{A.6})$$

Combining Eqs.(A.4), (A.5) and (A.6), we easily find $a_3 = a_4 = a_5 = 0$ and $a_6 = -\frac{1}{2}a_2$. So, Eq.(A.1) is rewritten as

$$\mathcal{H}_{RN}(m, Q) = a_1 r_h^2 + a_2 m r_h - \frac{1}{2}a_2 Q^2. \quad (\text{A.7})$$

Now, reducing the Reissner-Nodström black hole to the Schwarzschild black hole (namely, taking $Q = 0$) yields

$$\mathcal{H}_{RN}(m, Q) = 4a_1 m^2 + 2a_2 m^2. \quad (\text{A.8})$$

From Eq.(A.8), we can easily find both the dimensions corresponding to the constants a_1 and a_2 is the square of the mass m . Choosing $a_1 = 0$, we can obtain Eq.(2.5).

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